12/7/2015 الاثنين م. شهاب سكشن $\boxed{8}$

FFT :-

DIT — DIF

Desrimination in time — Descrimination in Prequency

even - odd — $n/2$, $n/2$

Properties:-

$$W_N^{K+N} = W_N^K \; ; \; W_N^{K+\frac{N}{2}} = -W_N^K \; ; \; W_N^2 = W_{N/2}$$

Given $x(n)$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad (\text{big-o } N^2)$$

$$= \sum_{\text{even}} x(n)\, W_N^{kn} + \sum_{\text{odd}} x(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \underbrace{x(2m)}_{f_1(m)} W_N^{2km} + \sum_{m=0}^{\frac{N}{2}-1} \underbrace{x(2m+1)}_{f_2(m)} W_N^{2km+k} \qquad \left(\text{big-0 } 2\left(\frac{N}{2}\right)^2 = \frac{2N^2}{4} = \frac{N^2}{2}\right.$$

$\frac{N}{2}$ DFT $\qquad$ $\frac{N}{2}$ DFT

$$= \sum_{m=0}^{\frac{N}{2}-1} x(2m)\left(W_{N/2}\right)^{km} + W_N^K \sum_{m=0}^{\frac{N}{2}-1} x(2m+1)\, W_{N/2}^{km}$$

$F_1(K) \Rightarrow \frac{N}{2}$ DFT ; $F_1(K) = F_1(K+\frac{N}{2})$

$F_2(K) \Rightarrow \frac{N}{2}$ DFT ; $F_2(K) = F_2(K+\frac{N}{2})$

$$X(K) = F_1(K) + W_N^K\, F_2(K) \; , \; K = 0 \text{ — } \frac{N}{2}-1$$

$$X(K) = F_1(K) + \underset{-W_N^K}{\cancel{W_N^{K+\frac{N}{2}}}}\, F_2(K) \; , \; K = 0 \text{ — } \frac{N}{2}-1$$

$$\left.\begin{array}{l} F_1(K) \\ F_2(K) \\ W_N^n \end{array}\right\} K = 0 \longrightarrow \frac{w}{2} - 1$$

$$f_1(m) \xrightarrow{DFT} F_1(K)$$

$f_1(2m)$ $\quad$ $f_1(2m+1)$

$$F_1(K) = F_{11}(K) + W_{N/2}^{K} F_{12}(K)$$

$$F_1(K) = F_{11}(K) - W_{N/2}^{K} F_{12}(K)$$

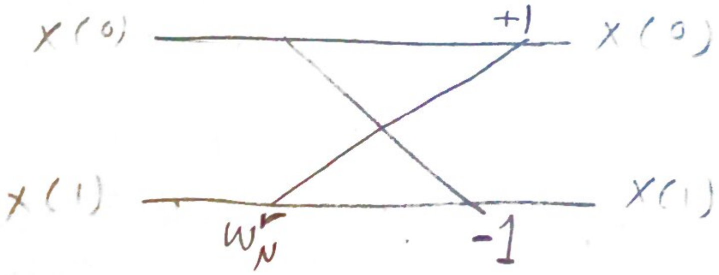
$\frac{N}{4}$ sequence

$$x(n) = \{ x(0), \; x(1) \}$$

$$X(K) = \sum_{n=0}^{1} x(n) W_2^{nK}$$

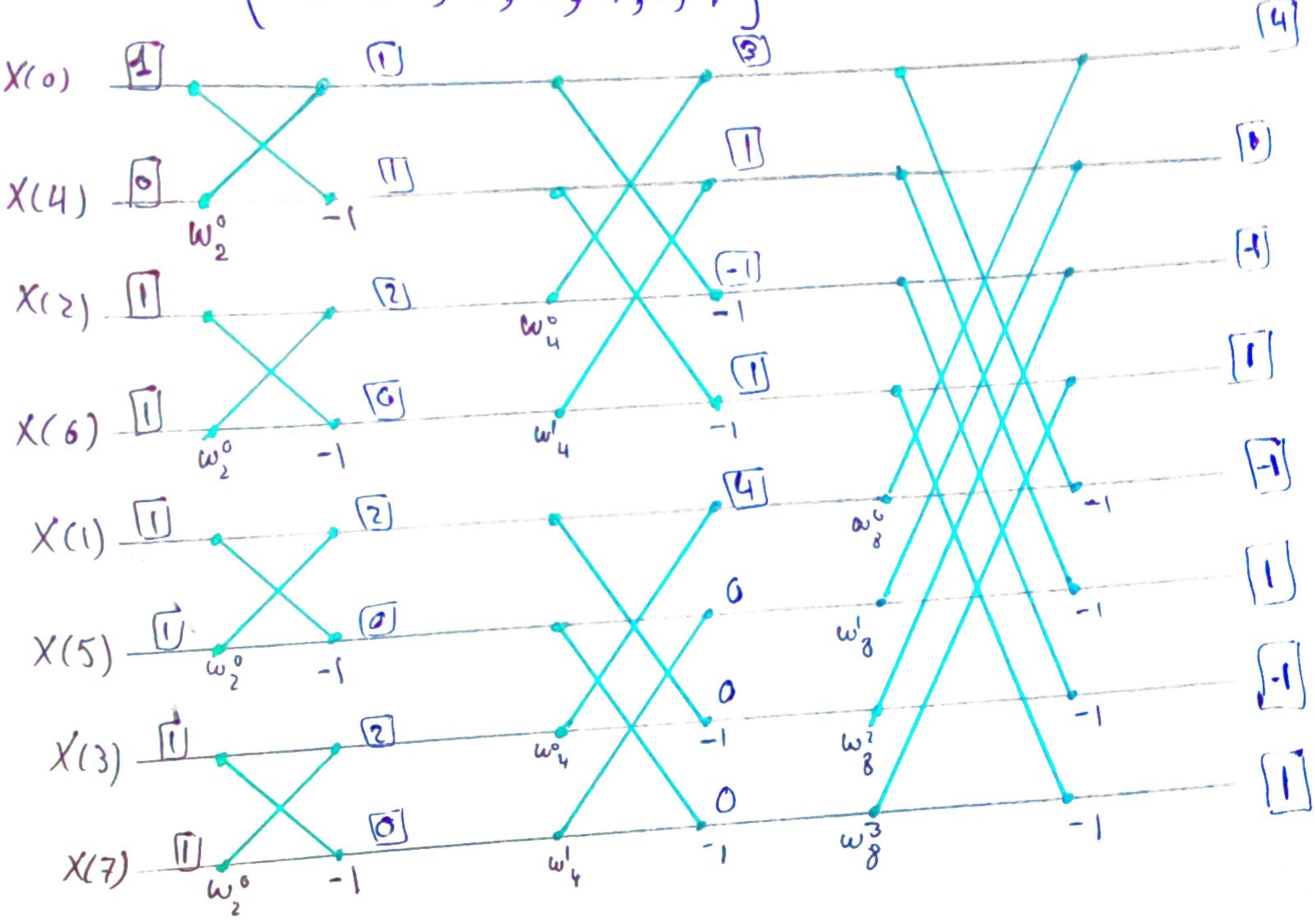
$$X(0) = x(0) + x(1)$$

$$X(1) = x(0) + \underset{-1}{\cancel{W_2^1}} x(1)$$



⟹ Turn Over

Ex: $x(n) = \{1, 1, 1, 1, 0, 1, 1, 1\}$


x(0)
1
x(4)
0
x(2)
1
x(6)
1
x(1)
1
x(5)
1
x(3)
1
x(7)
1
W_2^0
W_2^0
W_2^0
W_2^0
-1
-1
-1
-1
1
1
2
0
2
0
2
0
W_4^0
W_4^1
W_4^0
W_4^1
-1
-1
-1
-1
3
1
-1
1
4
0
0
0
W_8^0
W_8^1
W_8^2
W_8^3
-1
-1
-1
-1
4
0
-1
1
-1
1
-1
1

$r \to 0 — \frac{N}{2} - 1$